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Fitting Stationary Linear and Nonlinear Time Series Models to Nigerian Rainfall Data

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Abstract

Several time series modeling techniques assume linear relationship among the variables. However in some situations, variations among data are irregular and difficult to be accurately modeled. Linear relationship and their arrangements for describing the behavior of such data are inadequate. Since many real life data such as rainfall are nonlinear, thus there is need to investigate which models can best capture linear and nonlinear data. This paper examined the performances of the following nonlinear time series model: Self Exiting Threshold Autoregressive (SETAR), Smooth Transition Autoregressive (STAR) and Logistic Smooth Transition Autoregressive (LSTAR) models in fitting general classes of linear and nonlinear autoregressive cases at different sample sizes. The relative performances of the models were examined within the context of stationarity, and compared with linear Autoregressive (AR). The LSTAR was the best as sample size increased for different nonlinear autoregressive functions except in polynomial function where SETAR models out-performed others. The performances of the four fitted models increased as sample sizes increased. The application of aforementioned models was demonstrated on the monthly rainfall records of 1974-2013 in Nigeria. SETAR model

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fitted best to the data and LSTAR was the best when the data was transformed to nonlinear.

Keywords: Stationary, Linear, Nonlinear, Time Series, Nigerian Rainfall Data

Introduction

A time series is a sequence of data points, measured typically at successive points at uniform time intervals. Time series data is an array of time and numbers. In agriculture, records of annual crop and livestock production, and export and import sales etc. are recorded. While in ecology, records of the abundance of animal species are considered. The list of areas in which time series are studied is virtually endless. In meteorology, observation of variables such as temperatures, wind speed, relative humidity, rainfall, etc. were taken in different units of time. Of all the meteorological variables, rainfall is one of the most complex and difficulty elements to understand and model owing to its wide range of variability both in time and scale (French *et al.,* 1992; Akeyede *et al*, 2015). The complexity in the processes of rainfall generation and ever changing climatic conditions has made its quantitative prediction a difficult task (Hung et al., 2008; Abdulkadir *et al*., 2012). The behavioral pattern existing within the rainfall record is highly non-linear. Thus, sophisticated modeling tools are required. Time series analysis is generally used to model the stochastic mechanism that gives rise to observation series and to forecast the future values of a series based on the previous information. Usually, traditional time series analysis assumed linearity and stationarity among the variables. However, there have been great concerns in understanding the nonlinear and non-stationary time series models in several practical issues. The reason is that many real world problems do not satisfy the assumptions of linearity and/or stationarity. For Instance, the great complexities and variability that exist within the meteorological data requires further research beyond approximate linearity. Therefore, there is strong need to explore the theory and applications for nonlinear models in modeling meteorological (such as rainfall) records. Generally, time series analysis has a number of nonlinear features such as cycles, asymmetries, bursts, jumps, chaos, thresholds, heteroskedasticity, etc. The type of models that can be cast into this form are presented by Tong (1990), Granger and Terasvirta (1993), Franse and van Dijk (2000) and Tsay (2010). This study therefore considers some linear and nonlinear time series models and investigates the performance of these models in fitting linear, trigonometry, exponential and polynomial forms of autoregressive (AR) function on rainfall records. The goodness of fit for each model with information criteria was also considered in detail. A simulation study was carried out to verify the finite sample properties of the models for stationary data. The relative performances of each model were assessed using mean square error (MSE) and Alkaike Information Criteria (AIC).

The main aim of this study, therefore, is to suggest simple linear and nonlinear models stated earlier that can be fitted to rainfall data generated from general classes of linear and nonlinear second order autoregressive model. Its performance in finite sample cases was evaluated by simulation.

Methodology

Performance of autoregressive models such as SETAR, STAR and LSTAR were investigated on different classes of linear and nonlinear autoregressive time series by testing it on rainfall record obtained from Nigerian Meteorological Agency. The following describes all the criteria adopted in this present study

Self-Exciting Threshold Autoregressive (SETAR) Model

The threshold autoregressive (TAR) models are extension of autoregressive models that permit changing of parameters in the model according to the value of an exogenous threshold variable S_{t-d}. If the past value of means $S_{t-d} = Y_{t-d}$ is substituted, then it is called Self-Exciting Threshold Autoregressive model (SETAR). Some simple cases that are considered in this study are shown in Equations 1 and 2.

TAR Model

$$y_t = \begin{cases} \emptyset_0^1 + \emptyset_1^1 Y_{t-1} + \emptyset_2^1 Y_{t-2} + e_t^1 \, if \, S_{t-d} \le r \\ \emptyset_0^2 + \emptyset_1^2 Y_{t-1} + \emptyset_2^2 Y_{t-2} + e_t^2 \, if \, S_{t-d} > r \end{cases} \quad (1)$$

SETAR Model

$$y_t = \begin{cases} \emptyset_0^1 + \emptyset_1^1 Y_{t-1} + \emptyset_2^1 Y_{t-2} + e_t^1 \, if \, Y_{t-d} \le r \\ \emptyset_0^2 + \emptyset_1^2 Y_{t-1} + \emptyset_2^2 Y_{t-2} + e_t^2 \, if \, Y_{t-d} > r \end{cases} \quad (2)$$

Where d is the delay parameter and r is threshold value that initiate the changes between two different regimes. The threshold parameters satisfy the innovation within the ith regime e_t^i is a sequence of identically independent normal random variables with zero mean and constant variance $\sigma_i^2 < \infty (i = 1, 2)$. The overall process Yt, is non-linear when there are at least two regimes with different linear models. The simplest class of TAR models is the Self-Exciting Threshold Autoregressive (SETAR) models of the order p that was introduced by Tong (1983) and specified to order 2 as in Equation 2. The popularity of SETAR models is due to their relatively simplicity to specify, estimate and interpret as compared to many other nonlinear time series models in existence.

Smooth Transition AR (STAR) Model

The conditional mean equation is not continuous in the SETAR model as one of its critics. The thresholds (rj) are the discontinuity points of the conditional mean function μt. In an attempt to resolve this criticism, researchers proposed smooth TAR models (Chan and Tong, 1986; Terasvirta, 1994). A time series Yt follows a 2-regime STAR (p) model of the form

$$Y_t = c_0 + \sum_{i=1}^{p} \emptyset_{0.i} Y_{t-i} + F\left(\frac{Y_{t-d-\Delta}}{s}\right)\left(c_1 + \sum_{i=1}^{p} \emptyset_{1.i} Y_{t-i}\right) + e_t \quad (3)$$

d = delay parameter Δ and s are parameters representing the location and scale of model transition, and F(·) is a smooth transition function. Practically, F(·) often assumes one of three forms such as logistic, exponential or cumulative distribution function. The conditional mean of a STAR model is a weighted linear combination between Equations 4 and 5.

$$\mu_{1t} = c_0 + \sum_{i=1}^{p} \emptyset_{0.i} Y_{t-i} \quad (4)$$

$$\mu_{2t} = (c_0 + c_1) + \sum_{i=1}^{p} (\emptyset_{0.i} + \emptyset_{1.i}) Y_{t-i} \quad (5)$$

Equations 4 and 5 determine the properties of STAR model whose prerequisite for the stationary is that all zeros of both AR polynomials are outside the unit circle. In STAR model, the conditional mean function is differentiable as compared to TAR model. Although the transition parameters Δ and s of a STAR model are difficult to estimate is some cases. However, most empirical studies indicate that standard errors of $\Delta$and s are quite large, resulting in *t* ratios of about one (1.0) (Terasvirta, 1994). This uncertainty result into various complications in interpreting the results of STAR model.

Logistic Smooth Transition Autoregressive (LSTAR) Model
General model for a logistic smooth transition autoregressive model of order p [LSTAR(P)] model is as given in Equation 6.

$$Y_t = F(\gamma, c; Y_{t-d}) = (1 + exp - \{\gamma(Y_{t-d} - d)\})^{-1} \quad (6)$$

The coefficient $\gamma, \gamma > 0$ is the smoothness parameter and the scalar c is the location parameter and d is known as the delay parameter. Note, the variable $d > 0$ in model.

Criteria for Assessment of the Study

The goodness of fit for each model was assessed using common two criteria in time series, Mean square error and AIC. The model with lowest criteria is the best among the models for the simulated data.

Alkaike Information Criteria
The criteria to determine the best model of autoregressive process are likelihood-based. For instance, the well-known Akaike information criterion (AIC) (Akaike, 1973 cited by Tsay, 2010) is defined in Equation 7.

$$\text{AIC} = -\frac{2}{n} ln\ (\text{likelihood}) + \frac{2}{n}(\text{number of parameters}) \quad (7)$$

Where the likelihood function is evaluated at the maximum-likelihood estimates and *n* is the sample size.
Mean Squared Error
The values of mean squared error (MSE) measure the average of the squares of the "errors", that is, the difference between the estimator and what is estimated. If $\hat{Y}$ is a vector of estimated series, and Y is the vector of the true values, thus the estimated MSE is given as Equation 8

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\bar{Y} - Y_i)^2 \quad (8)$$

Selection Rule

The MSE and AIC for n = 50, 70, 100, 130, 150, 180, 200, 250, 300 and 400 for each case models were computed and the specific model that has the minimum criteria values was selected as the best among others. In this analysis, only the 2nd order of autoregressive model was considered in each case.

Models Selected For Simulation

The data are generated from several linear and nonlinear 2nd orders of general classes of autoregressive models expressed as given below. Each of the models is subjected to 1000 replication simulation at different sample sizes for stationary data structure.

Model I: AR(2): $Y_{ti} = 0.3Y_{ti-1} - 0.6Y_{ti-2} + e_t$

Model II: TR(2): $Y_{ti} = 0.3\sin(Y_{ti-1}) - 0.6\cos(Y_{ti-2}) + e_t$

Model III: EX(2): $Y_{ti} = 0.3Y_{ti-2} + \exp(-0.6Y_{ti-2}) + e_t$

Model IV: PL(2): $Y_t = 0.3Y_{t-1}^2 - 0.6Y_{t-2} + e_t$

$Y_{ti} \sim N(0,1) and e_{ti} \sim N(0,1)$ *for stationary series and* $Y_{ti} \sim N(2000,20) and e_{ti} \sim N(1000,10)$,

$t = 1,2, \ldots 50, 150$ *and* $300.\ i = 1,2, \ldots, 1000$

The models I, II, III and IV are linear (AR), trigonometry (TR), exponential (EX) and polynomial (PL) autoregressive functions respectively with coefficients of Y_{t-1} being 0.3 and Y_{t-2} being -0.6. The simulation studies investigate the performance of SETAR, STAR and LSTAR models for fitting different general classes of linear and nonlinear autoregressive time series stated above. Effect of sample size and the stationarity of the models were examined on each of the general linear and nonlinear data simulated.

However, innovation (error), e_t is often specified as independent and identically normally distributed in autoregressive modeling. The normal error implies that the stationary time series is a normal process. This indicates that any finite set of time series records are jointly normal. For example, the pair (Y_1, Y_2) and (Y_1, Y_2, Y_3) has bivariate and trivariate normal distribution respectively. This thus forms one of the basic assumptions of stationary data. However, the data will be generated under white noise assumption of stationarity and when the stationarity assumption is violated for order of past responses and random error terms to see behavior of the models in each case. In this study, one thousand (1000) replications were adopted to ensure the stability of the models estimations at different combinations of sample size (n) and models' types. The white noise assumption of the error term was also observed to make the data simulated be stationary. Rainfall data simulated were fitted to each of the model as shown in the goodness of fit model in Tables 1-4. Each of the created data were replicated 1000 times using tsDyn Package in R software

Results and Discussion

The performances of the fitted models on the basis of the two criteria of assessment were displayed in Tables 1-4. Table 1 shows the goodness of fit test for the four models (AR, SETAR, STAR and LSTAR) for model I (linear function) as described in Section 2.2.1 with the average values of MSE and AIC of 1000 replication simulated from each model at various sample sizes. The results obtained for MSE and AIC were graphically represented in Figures 1a and 1b respectively. The best fit to model I is AR having the least values of both MSE and AIC at sample size n = 50 followed by STAR model. However, with increase in sample size (see Table 1) the performance of the four fitted models increase as the magnitude of MSE and AIC keep decreasing.

Table 1. Performances of the Fitted Models on the Basis of Mean Square Error and AIC Criteria for model I

Sample	MSE				AIC			
Size(n)	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.0034	1.0704	1.0950	1.1162	1.2435	2.2551	14.9486	15.3815
80	1.0007	1.0632	1.0743	1.0769	1.0823	1.7752	10.3405	15.4199
100	0.9781	0.9482	0.9844	1.0341	0.6519	0.5120	5.3006	13.4578
130	0.9000	0.9326	0.9128	1.0160	-1.9453	0.0118	-0.3022	13.1352
150	0.8841	0.9265	0.9036	1.0067	-2.0358	-0.1791	-1.5080	12.5703
180	0.8378	0.9124	0.9223	1.0010	-3.0098	-0.3771	-1.6924	11.2028
200	0.8316	0.9021	0.8507	0.8999	-3.5045	-0.4642	-2.3415	9.0534
250	0.8127	0.8788	0.8568	0.8820	-4.0992	-0.7922	-3.1600	8.6308
300	0.8108	0.8502	0.8299	0.8439	-4.1467	-0.8058	-3.6287	8.5844
400	0.8076	0.7704	0.8081	0.8238	-5.1515	-1.0361	-4.9978	2.4269

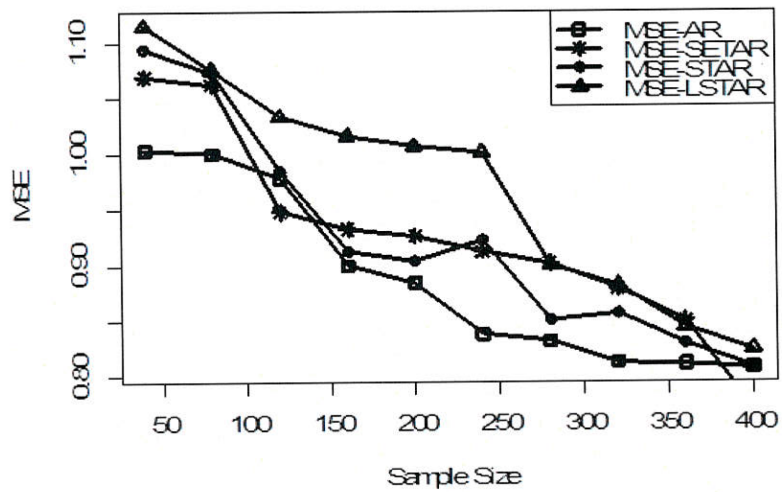

Figure 1a. MSE of the fitted models on model I

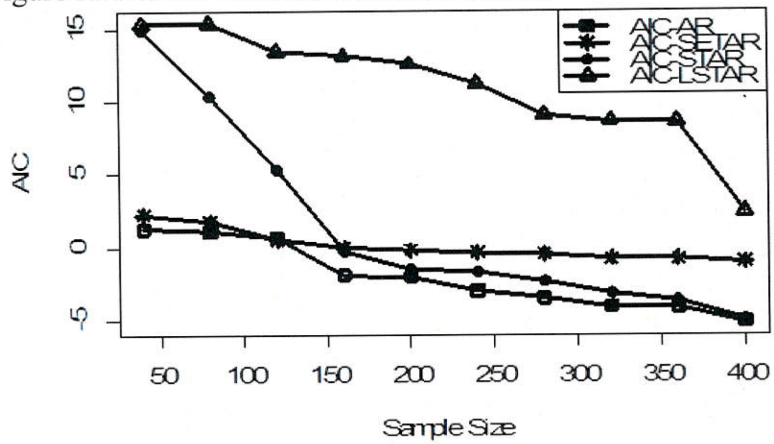

Figure 1b. AIC of the Fitted Models on Model I

More so, the best fit to model II (trigonometric function) is SETAR having the least values of both MSE and AIC at sample size n = 50. With increase in sample size (see Table 2) the performance of the four other fitted models increase as the magnitude of MSE and AIC keep decreasing. This indicates that the models performed excellently with increase in sample size.

Table 2. Performances of the Fitted Models on the Basis of Mean Square Error and AIC Criteria for model 2

Sample	MSE				AIC			
Size(n)	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.8778	1.0856	1.2555	1.1032	39.3347	25.1926	34.9504	27.7112
80	1.2878	1.0419	1.1979	1.1038	32.0152	22.1599	25.8792	25.0232
100	1.2410	1.0201	1.0322	1.1032	24.5778	17.8963	20.0044	19.6159
130	1.1213	1.0013	1.0253	1.0252	20.7725	13.6752	16.3612	14.3609
150	1.0942	0.9954	1.0195	0.9935	20.5946	11.9736	12.7373	12.8144
180	1.0770	0.9837	1.0016	0.9838	13.8477	11.8879	11.9715	11.7373
200	1.0211	0.9683	0.9935	0.9008	9.9183	10.6524	10.9698	6.6653
250	1.0123	0.9399	0.9871	0.8618	9.7873	8.5465	8.8975	6.4006
300	0.9988	0.9341	0.9838	0.8469	9.6550	6.5681	5.6446	3.0428
400	0.9119	0.9106	0.8472	0.8401	7.5899	4.6985	5.5609	1.1264

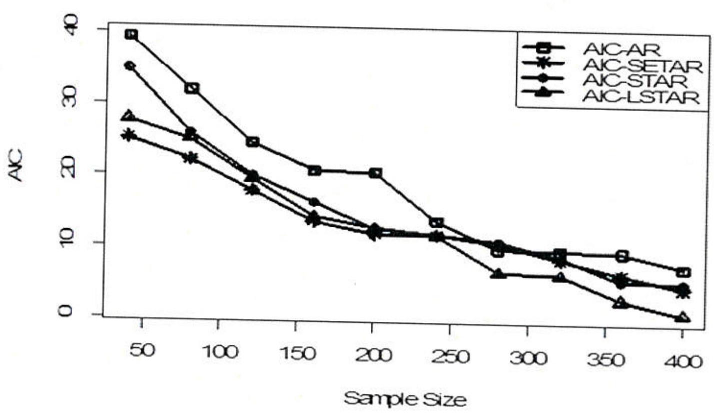

Figure 2b. AIC of the Fitted Models on Model II

From the results in Figures 2a and 2b, it was observed that LSTAR is fitted best to trigonometric function at sample sizes below 200 based but LSTAR is the best at sample size above 200 based on the two criteria. Meanwhile STAR competes well with SETAR as sample size increases. In model III (exponential function), Figures 3a and 3bindicated that STAR and LSTAR (see Table 3) performed equally at sample size below 200 but LSTAR supersede the other three models as sample size increases and fitted best to the exponential function at large sample sizes. For model IV (polynomial function), figures 4a and 4b showed that the best model is SETAR followed by STAR at sample sizes below 300 and LSTAR as sample size increases (see Table 4).

Table 3. Performances of the Fitted Models on the Basis of Mean Square Error and AIC Criteria for model 3

Sample	MSE				AIC			
Size(n)	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.5195	1.2044	1.5001	1.0843	29.7012	29.8014	15.1502	17.2593
80	1.3350	1.0192	1.0173	1.0045	23.133	20.2449	10.7939	10.7939
100	1.2870	1.0074	1.0044	0.9794	18.4658	14.855	10.7571	10.6242
130	1.2570	0.9973	0.9968	0.9735	17.1489	11.9369	7.8047	7.7358
150	1.1998	0.9802	0.9794	0.9479	15.8781	8.5940	7.7897	7.7009
180	1.1340	0.9744	0.9741	0.9053	15.518	8.0272	7.3729	4.9094
200	1.0932	0.9645	0.9732	0.8705	14.4622	6.4697	5.6874	0.3905
250	1.0390	0.9593	0.9391	0.8555	13.0971	5.8417	-4.3047	-6.8908
300	1.0206	0.9488	0.9238	0.8345	12.901	4.9595	-5.3248	-7.5132
400	0.8925	0.9259	0.9034	0.8054	12.5988	3.8493	-9.3038	-10.6848

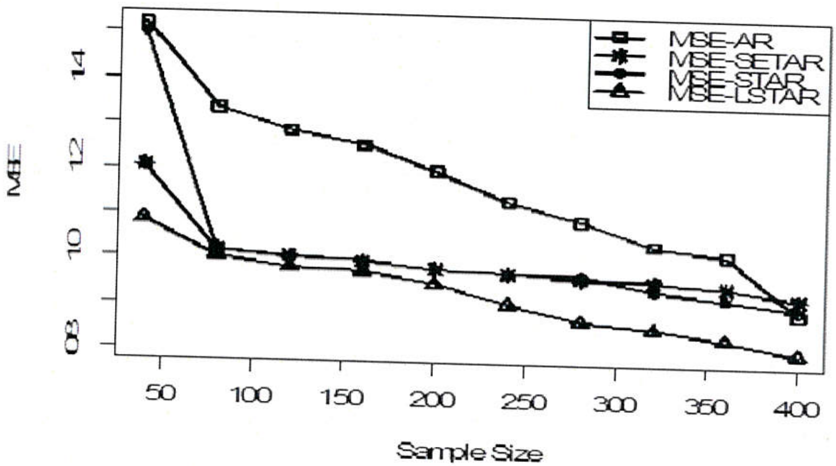

Figure 3a. MSE of the Fitted Models on Model III

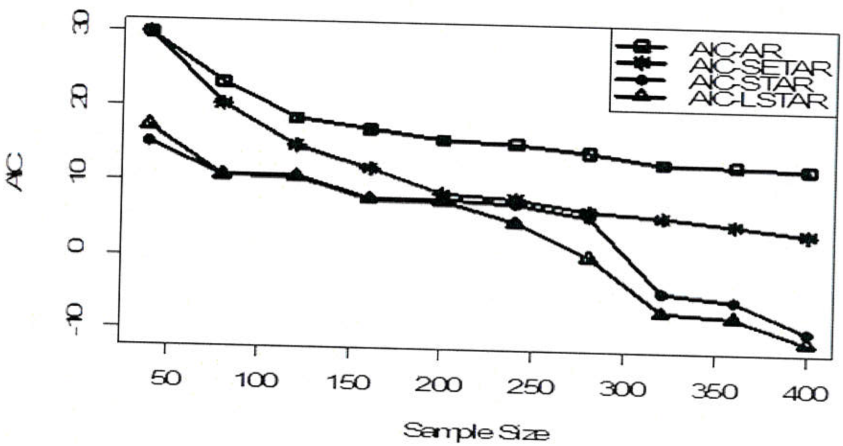

Figure 3b. AIC of the Fitted Models on Model III

Table 4. Performances of the Fitted Models on the Basis of Mean Square Error and Residual Variance Criterion for model 4

Sample	MSE				AIC			
Size(n)	AR	SETAR	STAR	LSTAR	AR	SETAR	STAR	LSTAR
50	1.5399	1.1077	1.5432	1.7703	161.2003	14.3851	50.9901	85.9526
80	1.5342	1.0389	1.3453	1.5884	159.8064	9.8809	49.8896	84.5517
100	1.5231	0.9812	1.3399	1.5007	158.0353	9.5891	32.481	80.3441
130	1.5134	0.9758	1.2253	1.4914	156.004	9.5891	32.4134	76.6701
150	1.5132	0.9757	1.2252	1.3844	154.5532	8.3157	32.4134	74.5517
180	1.4213	0.9492	1.2252	1.3833	50.8064	6.6123	32.4134	72.3378
200	1.3399	0.9279	1.2252	1.1194	47.0134	-1.6793	32.4134	30.6764
250	1.2350	0.8326	1.2248	1.0646	40.8526	-5.8224	32.4134	30.6619
300	1.1392	0.8202	1.1266	1.0115	30.2822	-9.4363	17.9162	12.2683
400	1.1222	0.7461	1.1062	1.0106	24.7411	-9.8073	14.0722	10.8421

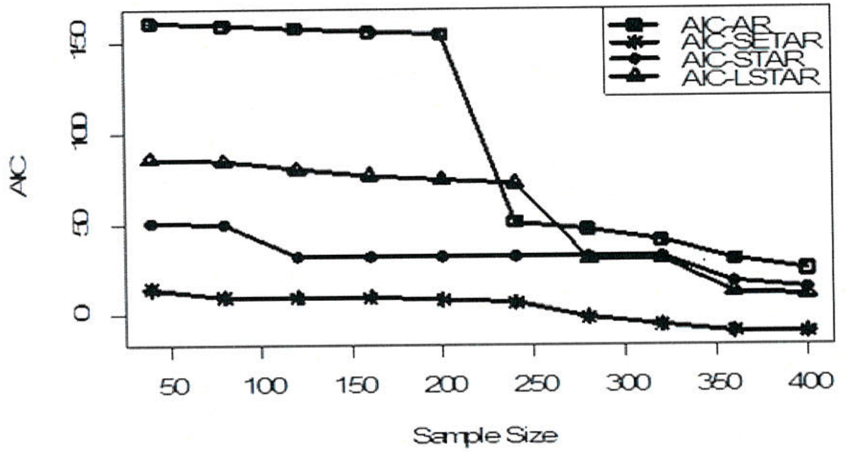

Figure 4a. AIC of the Fitted Models on Model IV

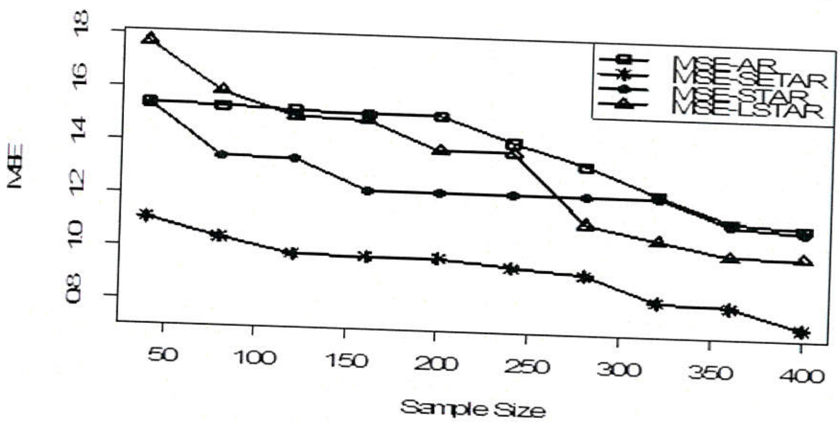

Figure 4b. MSE of the Fitted Models on Model IV

Application of the Fitted Models on Rainfall Data

The four models were fitted on monthly rainfall records of (1974–2013) obtained from Nigeria Meteorological Agency (NIMET), Lagos State Nigeria. Prior to fitting a nonlinear time series model to the set of rainfall record, the nonlinearity characteristics of the data are detected using recommended tests by researchers to distinguish between linear and nonlinear data sets. For instance Subba and Gabr (1980) and Hunnich (1982) used the bi spectrum test. In this test, the square modulus of normalized bi spectrum is constant for a linear time series. The hypothesis is based on the non-centrality of parameters of the marginal distribution of the square moduli. Yuan (2000) modified the Hunnich's test in such a way that the parameter being tested under the null hypothesis is no longer but the location parameters, such as the mean or variance. Furthermore, once a model is selected, there is need to check if the data is linear. Therefore, good statistical and diagnostic tests are needed to determine the nonlinearity in time series data. However in this research, two tests were used to detect whether the rainfall data is nonlinear or linear. The tests are Keenan and Tsay F-tests. Both tests are based on time domain. These have been used by many researchers for the detection of nonlinearity in time series data (Keenan, 1985; Tsay, 1986). The data was transformed using logarithmic transformation to ensure nonlinearity test and the results are shown in Table 5. This shows that the null hypothesis of nonlinearity was rejected for the rainfall data before being transformed but accepted after being transformed using the two statistical approaches. Table 6 shows the performance of the models indicated that SETAR is the best to fit the rainfall data followed by LSTAR. However, when the data is transformed to make it nonlinear, LSTAR performs better than others based on the three criteria (of nonlinearity, MSE and AIC).

Table 5 Test of Nonlinearity on Monthly Rainfall in Nigeria (1974-2013)

Nonlinearity	Real Data				Transformed Data			
Test	Test-Stat	DF	p-value	Decision	Test-Stat	DF	p-value	Decision
Keenan	8.1645	24	0.0045	reject	1.4837	24	0.22390	accept
Tsay F	1.5340	24	0.0027	reject	1.7870	24	0.09424	accept

Table 6: Performances of the Fitted Models on Monthly Rainfall (1974-2013)

	Real Data		Transformed Data	
Model	MSE	AIC	MSE	AIC
AR	6022.00	5546.21	2.7770	1858.8900
SETAR	5873.27	4179.52	2.4041	435.0518
STAR	5998.32	4181.63	2.3913	434.7764
LSTAR	5873.29	4181.52	2.3839	432.9900

Conclusion

In this study, the performances of four nonlinear models were investigated on rainfall records due to its nonlinearity nature. The best model to fit linear autoregressive function is AR at different sample sizes of which LSTAR model out performed other models as the sample size increases with exception of polynomial function where SETAR model performed better than others. The three nonlinear models SETAR, LSTAR and STAR have closed performances in exponential autoregressive function as number of sample size increases based on MSE and AIC criteria. The performance of the four fitted models increases as sample size increases. Finally, it was observed that SETAR model fits best to the rainfall data and LSTAR was the best when the data is transformed to nonlinear. The results also showed that the null hypothesis of nonlinearity of rainfall records was rejected as real data but accepted as transformed data.

References

Abdulkadir, T.S., Salami, A.W. and Kareem, A.G. (2012). Artificial Neural Network Modeling of Rainfall of Ilorin, Kwara State, Nigeria, USEP-RICE, Journal of Research Information in Civil Engineering,Volume9(1), pp. 108-120.

Akaike, H. 1973. Maximum Likelihood Identification of Gaussian

Akeyede, I. Adeleke, B.L. and Yahya, W.B. (2015). On forecast Strength of some linear and nonlinear time series models for stationary data structure, American Journal of Mathematics and Statistics 2015, 5(4): 163-177 DOI: 10.5923/j.ajms.20150504.01

Auto-regressive Moving- Average Models. Biometrika, Volume 60, pp. 255–266.

Chan, K.S. and Tong, H. (1986). On Estimating Threshold in Autoregressive Models, Journal of Time Series Analysis, Volume 7, pp. 179-194.

Franses, P.H. and Van Dijk, D. (2000). Nonlinear Error-correction Models for Interest Ates in the Netherlands, in Barnett W.A., Hendry D.F., Hylleberg S.,Terasvirta T.,Tjøstheim D. and Wurtz A. H. (eds.), Nonlinear Econometric Modeling, Cambridge: Cambridge University Press.

French, M.N., Krajewski W.F. and Cuykendall, R.R. (1992). Rainfall Forecasting in Space and Time Using Neural Network, Journal of Hydrology, Volume 137, pp. 1-31

Granger, C. and Teräsvirta, T. (1993). Modeling Nonlinear Economic Relationships. Oxford University Press, New York.

Hung, N.Q., Babel M.S., Weesakul, S. and Tripathi N.K. (2008). An Artificial Neural Network Model for Rainfall Forecasting in Bangkok, Thailand, Hydrology and Earth System Sciences Discussion,Volume5, pp. 183-218.

Keenan, D.M. (1985). A Tukey Non-additivity-type Test for Time Series Nonlinearity. Biometrika, Volume 72(1), pp. 39-44.

Subba, R.T. and Gabr, M.M. (1980). A Test for Linearity of Stationary Time series. Journal of Time series Analysis, Volume 1(1), pp. 145-158.

Terasvirta, T. (1993). Testing Linearity against Smooth Transition Autoregressive Models. Biometrika, Volume 74, pp. 491-499.

Terasvirta, T. (1994). Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models. Journal of the American Statistical Association, Volume 89, pp. 208–18.

Tong, H. (1990). Nonlinear Time Series: A Dynamical System Approach, Oxford, Clarendon.

Tsay, R.S. (1986). Nonlinearity Tests for Time Series, Biometrika, Volume 73, pp. 461-466.

Tong, H. (1983). Threshold models in non-linear time series analysis, Lecture Notes in Statistics, No. 21, Heidelberg: Springer, 62-64.

Tsay, R.S. (2010). Analysis of Financial Time Series (3rd Edition), John Wiley & Sons, Inc., Publication, pp. 177 – 214.

Yuan, J. (2000). Testing Linearity for Stationary Time Series using the Sample Interquartile Range. Journal of Time Series Analysis, Volume 21(6), pp. 714-722.